

## The Hubble Parameter in Void Universe

*-Effect of the Peculiar Velocity-*

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We investigate the distance-redshift relation in the simple void model. As discussed by Moffat and Tatarski, if the observer stays at the center of the void, the observed Hubble parameter is not so different from the background Hubble parameter. However, if the position of observer is off center of the void, we must consider the peculiar velocity correction which is measured by the observed dipole anisotropy of cosmic microwave background. This peculiar velocity correction for the redshift is crucial to determine the Hubble parameter and we shall discuss this effect. Further the results of Turner et al by the N-body simulation will be also considered.

Recent observation suggests that Hubble parameter is large one, that is,  $80 \pm 17 \text{ km/sec/Mpc}$  (Freedman et al. 1994). The low Hubble universe, however, is favored since the small value of Hubble parameter is consistent with almost all observations except for that of the Hubble parameter itself (Bartlett et al. 1994). One of the theoretical bases for the possibility of smaller Hubble parameter than that determined by local observation is given by Turner, Cen and Ostriker (Turner et al. 1992). They performed very large scale N-body simulations and constructed the ensemble of universe filled with the galaxies which, roughly speaking, are defined by density peaks of collisionless particles. Then, one of those galaxies is identified as “our galaxy” and they investigate the relation between the distance of the other galaxies from our galaxy and the relative velocity with the correction about the peculiar velocity only of our galaxy. Their result suggests that the Hubble parameter determined by such observations has the scale dependent variance. In order to obtain the correct Hubble parameter, we need the observation of galaxies over the very wide region.

On the other hand, Moffat and Tatarski considered the void universe in which the observer is assumed to be

at the center of void and investigated the effect of the void on the Hubble parameter determined through the redshift and distance relation (Moffat & Tatarski 1994). Their result reveals that when the observer is at the center of void, the Hubble parameter is not so different from the true value as long as the observed region is smaller than the curvature radius within the void. This seems to contradict with the results of Turner et al.

In this paper, we investigate the void universe, but shall not restrict the position of the observer to be the center of the void. Our void model is more simplified one than that of Moffat and Tatarski, but will clarify the effect of the inhomogeneities on the observation of the Hubble parameter. We assume that the inside of the void is approximated by the Friedmann-Robertson-Walker (FRW) universe with the present density parameter  $\Omega_0 < 1$  while the outside of the void is also the FRW universe but with  $\Omega_0 = 1$ . The boundary of the void can be ignored as long as we exist within the void and observe only inside of that. Here we will assume such a situation. Further we assume that the age of both inside and outside of the void is the same and hence the time coordinate is common cosmic time  $t$  to both the inside and

outside of the void. This assumption corresponds to the fact that the void structure comes from purely growing mode of the initial density perturbation since the density contrast between the inside and outside of the void vanishes as  $t \rightarrow 0$ , i.e., at the initial singularity.

The metric within the void is written as

$$ds^2 = -dt^2 + \frac{a_v^2(t)}{1 + (R_v/R_c)^2} dR_v^2 + a_v^2(t) R_v^2 dS^2, \quad (1)$$

where  $R_c$  is the comoving curvature radius and  $dS^2 = d\theta^2 + \sin^2\theta d\varphi^2$  is the line element on the unit sphere. We should note that the center of the void agrees with the origin  $R_v = 0$  and hence, as for the time coordinate  $t$ ,  $dS^2$  is common to the inside and outside of the void. As is well known, the scale factor  $a_v$  is given as the parametric form by the conformal time  $\eta$ ,

$$\frac{a_v}{a_{v0}} = \frac{\Omega_{v0}}{2(1 - \Omega_{v0})} (\cosh \eta - 1), \quad (2)$$

$$H_{v0}t = \frac{\Omega_{v0}}{2(1 - \Omega_{v0})^{3/2}} (\sinh \eta - \eta), \quad (3)$$

where  $H_{v0}$ ,  $a_{v0}$  and  $\Omega_{v0}$  are, respectively, the present Hubble parameter, the present scale factor and the present value of the density parameter, within the void.

On the other hand, we assume that the outside of the void is the flat FRW universe and hence its metric outside the void is given by

$$ds^2 = -dt^2 + a_b^2(t)(dR_b^2 + R_b^2 dS^2), \quad (4)$$

and the scale factor  $a_b$  is written as

$$\frac{a_b}{a_{b0}} = \left(\frac{9}{4} H_{b0}^2 t^2\right)^{1/3}, \quad (5)$$

where  $a_{b0}$  and  $H_{b0}$  are, respectively, the present scale factor and the present Hubble parameter, outside the void.

As discussed by Bartlett et al. (1994), the ratio,  $H_{v0}/H_{b0}$ , varies over the range  $3/2$  to  $1$  as  $\Omega_{v0}$  varies from  $0$  to  $1$ . Hence the maximum Hubble parameter within the void is at most  $3/2$  times the background Hubble parameter  $H_{b0}$ . However, it should be noted that  $H_{v0}$  is not observed directly. The observed Hubble parameter is determined through the relation between the distance and redshift with the correction about the peculiar velocity of both the observer and the observed source.

Here we define the peculiar velocity which is crucial to estimate the true redshift. Assuming that the cosmic microwave background (CMB) radiation is homogeneous and isotropic, the peculiar velocity is defined as that against the frame in which the CMB is observed to be isotropic. Since the comoving observer outside the void just observes the isotropic CMB, we first define the new radial coordinate  $\tilde{R}$  for inside of the void as  $a_v(t)R_v = a_b(t)\tilde{R}$ . It should be noted that the observer along  $\tilde{R}=\text{constant}$  curve looks just isotropic CMB. The transformation matrix is given by

$$d\tilde{t} = dt, \quad (6)$$

$$d\tilde{R} = \frac{a_v}{a_b}(H_v - H_b)R_v dt + \frac{a_v}{a_b}dR_v, \quad (7)$$

$$d\tilde{S}^2 = dS^2. \quad (8)$$

In the original coordinate (1), the comoving observer and comoving observed source move along  $R_v = \text{constant}$  lines and hence the components of those 4-velocities are given by the common  $u^\mu = (1, 0, 0, 0)$ . On the other hand, in the above tilde coordinate system, the components are given by

$$u^{\tilde{t}} = \frac{\partial \tilde{t}}{\partial t} u^t = 1, \quad (9)$$

$$u^{\tilde{R}} = \frac{\partial \tilde{R}}{\partial t} u^t = \frac{a_v}{a_b}(H_v - H_b)R_v, \quad (10)$$

$$u^{\tilde{\theta}} = 0 = u^{\tilde{\varphi}}, \quad (11)$$

and the radial component  $u^{\tilde{R}}$  corresponds to the peculiar velocity of the comoving observer in the void.

In order to obtain the relation between the redshift and the distance, it is sufficient to approximate the light ray by a null geodesic, i.e., to treat the propagation of light ray by the geometric optics (Misner, Thorn & Wheeler 1973). By virtue of the spherical symmetry of this system, without loss of generality, we focus only on the null geodesic within the equatorial plane  $\theta = \pi/2$ . The solution for the null geodesic tangent  $k^\mu$  is then given by

$$k^t = \frac{a_{v0}}{a_v(t)} \omega_{v0}, \quad (12)$$

$$k^{R_v} = \pm \frac{a_{v0}}{a_v^2(t)} \sqrt{[1 + (R_v/R_c)^2][\omega_{v0}^2 - (L_0/R_v)^2]}, \quad (13)$$

$$k^\varphi = \frac{a_{v0}L_0}{a_v^2(t)R_v^2}, \quad (14)$$

and  $k^\theta = 0$ . The radial trajectory of the null geodesic is obtained as

$$R_v = R_k(\eta) \equiv R_c \sqrt{F^2(\eta) - 1}, \quad (15)$$

with

$$F(\eta) = \sqrt{1 + \left(\frac{L_{v0}}{\omega_{v0}R_c}\right)^2} \cosh \left[ \cosh^{-1} \left\{ \sqrt{1 + \left(\frac{R_{v0}}{R_c}\right)^2} \right. \right. \\ \left. \left. / \sqrt{1 + \left(\frac{L_{v0}}{\omega_{v0}R_c}\right)^2} \right\} \pm (\eta - \eta_0) \right], \quad (16)$$

where  $R_{v0}$ ,  $L_{v0}$  and  $\omega_{v0}$  are the integration constants and  $\eta_0$  is the present conformal time. It should be noted that, at  $\eta = \eta_0$ ,  $(R_v, \varphi) = (R_{v0}, 0)$  and this corresponds to the position of the comoving observer at the moment of observation.  $L_{v0}$  is the conserved angular momentum of the light ray while,  $\omega_{v0}$  is the angular frequency of that for the comoving observer. Together with  $\omega_{v0}$ ,  $L_{v0}$

determines the angle  $\theta_k$  between the radial direction and the propagation direction of the light ray as, (see Fig.1),

$$\cos \theta_k = \mp \sqrt{1 - \left( \frac{L_{v0}}{\omega_{v0} R_{v0}} \right)^2}. \quad (17)$$

Next, we consider the effect of the peculiar velocity on the angular frequency of the light ray. The comoving observer (comoving observed source) detects (emits) the light ray  $k^\mu$  with the angular frequency,  $\omega_v \equiv -k_\mu u^\mu = -k_t$ . On the other hand, the observer and observed source moving along  $\tilde{R} = \text{constant}$  curve, have 4-velocity  $w^{\tilde{\mu}} = (1, 0, 0, 0)$  in the tilde coordinate and hence the angular frequency for those is given by

$$\begin{aligned} \omega_c &\equiv -k_{\tilde{\mu}} w^{\tilde{\mu}} = -k_{\tilde{t}} = \omega_v + k_{\tilde{R}} u^{\tilde{R}} \\ &= \omega_v + (H_v - H_b) R_v k_{R_v}. \end{aligned} \quad (18)$$

It should be noted that  $\omega_c$  corresponds to the angular frequency with the correction for the peculiar velocity. Observationally, we can consider the effect only of our own peculiar velocity and hence hereafter we focus on the quantities with the correction about the peculiar velocity only of the observer and those without any corrections for the peculiar velocity. Then we define the following two kinds of the redshift as

$$z = \frac{\omega_v}{\omega_{v0}} - 1, \quad \text{and} \quad z_{co} = \frac{\omega_v}{\omega_{co}} - 1, \quad (19)$$

where  $\omega_v$  is the angular frequency of the light ray at the observed source while  $\omega_{co}$  is given by

$$\omega_{co} = \omega_{v0} + (H_{v0} - H_{b0}) R_{v0} k_{R_v}(\eta_0). \quad (20)$$

Hence,  $z$  is the bear observed redshift and  $z_{co}$  is the redshift with the correction about the peculiar velocity only of the observer.

We shall employ the luminosity distance  $D_L$  as the distance measure between the observer and observed source. Here, the luminosity distance  $D_L$  is given by well-known relation in the FRW universe with (1) as

$$D_L = \frac{1}{H_{v0} q_{v0}^2} [z q_{v0} + (q_{v0} - 1)(-1 + \sqrt{2q_{v0}z + 1})], \quad (21)$$

where  $q_{v0} = \Omega_{v0}/2$ . It should be noted that the luminosity distance  $D_L$  is just the observed quantity which is determined by, for example, Tully-Fisher relation. Then, using  $D_L$ , we define the observed Hubble parameter  $H_{co}$  with the correction for the peculiar velocity only of the observer with the assumption that observers regard their own universe as the flat FRW space-time,

$$H_{co} = \frac{2}{D_L} [z_{co} + 1 - \sqrt{z_{co} + 1}]. \quad (22)$$

In fact, we can measure  $H_{co}$  instead of  $H_{b0}$  in the real observations. In Fig.2,  $H_{co}$  is depicted for  $\theta_k = 0, \pi/2$  and  $\pi$ . In this figure, the density parameter inside the

void,  $\Omega_{v0}$  is equal to 0.1 and the radial position of the observer is fixed as  $a_{v0} R_{v0} = 1 \times 10^{-2} H_{b0}^{-1} \sim 30 h_b^{-1} \text{Mpc}$ .

We find that, for  $H_{v0} D_L \ll 1$ ,  $H_{co}$  strongly depends on the observed direction along which the light ray propagates. This comes from the wrong peculiar velocity correction and it should be noted that the Hubble parameter defined by Turner et al. is a volume average of just  $H_{co}$ .

To understand the direction dependence of  $H_{co}$ , we investigate that only for  $H_{v0} D_L \ll 1$ . In this case,  $H_{co} \sim z_{co}/D_L \sim H_{v0}(z_{co}/z)$  and, assuming the case of  $\Omega_{v0} = 0.1$ , we obtain  $H_{v0}/H_{b0} - 1 \sim 0.35$ . Further, the distance,  $a_{v0} R_{v0}$ , of the observer from the center of the void is assumed to be less than about  $100 h_b^{-1} \text{Mpc}$ , i.e.,  $a_{v0} H_{b0} R_{v0} < 3 \times 10^{-2} \ll 1$ . Hence, we obtain

$$\begin{aligned} z_{co} &\sim H_{v0} D_L - 10^{-2} \times \frac{1}{\omega_{v0}} \\ &\times (H_{v0} D_L + 1) k_{R_v}(\eta_0) \left( \frac{R_{v0}}{100 \text{Mpc}} \right). \end{aligned} \quad (23)$$

Since  $a_{v0} R_c = H_{v0}^{-1} (1 - \Omega_{v0})^{-1/2} \sim H_{v0}^{-1}$ ,  $R_{v0}/R_c$  is much less than unity and hence we can see that  $k_{R_v}(\eta_0) \sim -\omega_{v0} \cos \theta_k$ . Then we get

$$\frac{H_{co}}{H_{b0}} \sim \frac{H_{v0}}{H_{b0}} + 10^{-2} \frac{1}{H_{v0} D_L} \cos \theta_k \left( \frac{R_{v0}}{100 \text{Mpc}} \right). \quad (24)$$

From the above equation, when the distance of the observer from the center of void is  $30 h_b^{-1} \text{Mpc}$  and when such an observer looks to the direction  $\theta_k = 0$  and the observed distance is  $D_L = 3 \times 10^{-3} H_{v0}^{-1} \sim 7 h_b^{-1} \text{Mpc}$ , the observer may estimate  $H_{co}$  to be factor two times larger than  $H_{b0}$ . On the other hand, if that observer looks to the opposite direction  $\theta_k = \pi$ , the observer may obtain almost vanishing  $H_{co}$ . This is just the dipole anisotropy due to the wrong correction for the peculiar velocity.

Here we shall consider the relation between our simple void model and the results by Turner et al. In our case, the averaged  $H_{co}$  agrees with  $H_{v0}$  as

$$\langle H_{co} \rangle = \frac{1}{\pi} \int_0^\pi d\theta_k H_{co} = H_{v0}. \quad (25)$$

It should be noted that we assume the uniform distribution of observed source, i.e., galaxy when we perform the above averaging. However, in the N-body simulation, the ‘‘galaxy’’ is not uniformly distributed in contrast with our model and the integral of the second term in R.H.S. of Eq.(24) may remain. Fig.1 shows an example in which the number of galaxies on the direction  $\theta_k = 0$  is larger than that on  $\theta_k = \pi$  direction. In such a case, the averaged  $H_{co}$  is greater than  $H_{v0}$ . Therefore it may be a reason why the variance of the Hubble parameter depends on the scale of the observational regions and there appears the large variance of the Hubble parameter in the small scale observation in the results of Turner et al. Of course, in order to confirm this expectation, the detailed investigation by the N-body simulation is needed (Gouda et al. 1995).

From the observational point of view, if the variance of Hubble parameter comes from the dipole anisotropy such as above, it is important to confirm the isotropy of Hubble parameter. Lauer and Postman reported the highly isotropic Hubble parameter by the rather large scale observation  $z \leq 0.05$  (Lauer & Postman 1992). Hence, even if we stay in the void, we are near the center of that. In the case that we stay near the center of the void, the observed Hubble parameter is  $H_{v0}$  and this varies over the range  $H_{b0}$  to  $1.5H_{b0}$ . Since this is not so large variance, we can find almost the same Hubble parameter as the background one. Of course, our model is too simple and more complicated situations may be imagined, which makes us to fail the true Hubble parameter. Hence further theoretical investigation should be continued and deeper observation over whole direction in the sky is very important.

Finally, we should comment on the effect of the void on the anisotropy of CMB. Here we shall assume that the CMB is completely isotropic at the last scattering surface and that the anisotropy is caused only by the effect of one void. The dipole anisotropy of CMB is about  $v/c$  where  $v$  is the peculiar velocity of the co-moving observer in the void and it is given roughly as  $(H_{v0} - H_{b0})a_{v0}R_{v0}$  by Eq.(10). If the density parameter inside the void is nearly zero, we obtain  $v \sim 1.5 \times 10^3(a_{v0}R_{v0}/100h_b^{-1}\text{Mpc})\text{km/sec}$ . Assuming that the observed dipole anisotropy comes from the peculiar velocity of our local group, that is estimated as about  $600\text{km/sec}$  (Smoot et al 1991). If we live in such a void, then our position is  $10h_b^{-1}\text{Mpc}$  apart from the center of the void. However our void considered here is nothing but a toy model and it should not be seriously considered.

The rather serious subject is the quadrupole or higher multi-pole anisotropies which come from the gravitational redshift. We consider the situation that the size of the void is sufficiently smaller than the horizon scale  $L$  of the background flat FRW universe and hence the Newtonian approximation is applicable. In this case, the metric is written as

$$ds^2 = -(1 - 2U)dt^2 + a_b^2(t)(1 + 2U)(dR^2 + R^2dS^2), \quad (26)$$

where  $|U| \ll 1$ . Further we assume the step-function-like density configuration,

$$\rho = \begin{cases} \rho_v(t) & R < R_{void} \\ \rho_b(t) & \text{otherwise} \end{cases} \quad (27)$$

where  $\rho_b$  corresponds to the critical density. Then the Newton potential  $U$  inside the void  $R < R_{void}$  is obtained as

$$U = 2\pi\delta\rho\ell^2 - \frac{2\pi}{3}\delta\rho(a_bR)^2, \quad (28)$$

where  $\ell \equiv a_bR_{void}$ . Here, since we consider the case in which  $\delta\rho \sim -\rho_b \sim -H_b^2 = -L^{-2}$ , we see that  $\delta\rho\ell^2 \sim \kappa^2 \equiv (\ell/L)^2 \ll 1$ . Thus we can roughly estimate the Newtonian potential as  $U \sim \kappa^2 - \kappa^2(a_bR/\ell)^2$ ,  $\partial_t U \sim H_b U$  and  $\partial_r U \sim \kappa^2(a_b/\ell)^2 R$ . Here we shall estimate the Sachs-Wolfe effect on the CMB by the above Newtonian potential. The anisotropy of CMB is expressed by the integrated brightness temperature perturbation  $\Theta$  and the equation for  $\Theta$  is written as

$$\frac{d}{dt}(\Theta - U) \equiv \left(\partial_t + \frac{\gamma^i}{a_b}\partial_i\right)(\Theta - U) = -2\partial_t U, \quad (29)$$

where  $\gamma^i$  is the direction cosine of the photon (Kodama & Sasaki 1986). Then, the difference between the two opposite radial directions is roughly estimated as

$$\frac{\Delta T}{T} = 2\left(\int dt\partial_t U|_{\theta_k=0} - \int dt\partial_t U|_{\theta_k=\pi}\right) \sim \kappa^3\left(\frac{R_o}{\ell}\right), \quad (30)$$

where  $R_o$  denotes the radial position of the observer and the integration is performed along the path of the light ray. It should be noted that the above result is consistent with the analysis by Meszaros (1994) for the case that the position of the observer is outside of the void. Hence if we live in the  $100h_b^{-1}\text{Mpc}$  scale void, since  $\kappa^3 \sim 4 \times 10^{-5}$ , the higher multi-pole anisotropy of CMB by the such a void does not conflict with *COBE* results (Smoot et al. 1992). However, this estimate is so rough that we need more detailed investigation and this is in progress.

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#### REFERENCES

- Freedman W.L. et al., 1994, *Nature*, **371**, 757  
 Bartlett J.G., Blanchard A., Silk J., Turner M., 1994, *FERMILAB-Pub-94/173-A* (1994)  
 Turner E.L., Cen R., Ostriker J.P., 1992, *Astron. J.*, **103**, 1427  
 Moffat J.W., Tatarski D.C., 1994, preprint, UTPT-94-19  
 Misner C.W., Thorne K.S., Wheeler J.A., 1973, *Gravitation*(Freeman)

The Hubble parameter  $H_{co}$  with the correction for the peculiar velocity only of the observer is plotted against the luminosity distance  $D_L$  for various direction. The density parameter  $\Omega_{v0}$  within the void is 0.1.

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